Exchanging Variable Annuities: An Optional Test for Suitability

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ABSTRACT

In this paper we offer a novel method of assessing whether exchanging one variable annuity (VA) policy for another, destroys or adds value from a purely economic perspective. We do this by decomposing the policy into a portfolio of financial options and then use an option pricing model to compute the difference in aggregate value between the embedded options in the new and old VA. Our paper illustrates this approach with a variety of case studies using a software implementation that is available on the journal's website. We also draw some general conclusions about the conditions under which a VA exchange is likely to be suitable. Overall, we believe that our methodology is a non-biased and objective technique for mitigating some of the long-standing concerns about excessive churning of VAs.

MOTIVATION AND INTRODUCTION

According to statistics compiled by the National Association of Variable Annuities (source: NAVA Outlook March/April 2003) during the calendar year 2002, approximately $113.7 billion in variable annuity sales were recorded by the 25 largest annuity writers in the United States. Of this sum, it is estimated that only $30.7 billion represented *net flows*, which implies that close to 73% of sales were effectively exchanges from one product to another. One can only speculate on whether these transactions -- which sometimes come under the label of Section 1035 exchanges[3] -- added economic value to the policyholder.

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[3] Strickly speaking, the term "section 1035 exchange" only covers annuities that are being exchanged outside of qualified plans. When the VA funds a qualified plan, the exchange is treated as a transfer between vendors. We will use the term exchange generically to cover both situations.

This trend has been noticeable for years, and perhaps as a reaction to this immense flow of funds from one company to another, the National Association of Securities Dealers (NASD) issued an investor alert in early 2001 regarding the wisdom behind exchanging one variable annuity for another[4].

Motivated by the substantial volume of this market and the flood of regulatory and consumer interest surrounding the question of suitability, the purpose of this paper is to provide a framework for analyzing the costs and benefits from switching one variable annuity policy for another. The essence of our methodology is to decompose the VA policy into a portfolio of financial guarantees and then use an option pricing model to compute the difference in aggregate value between the embedded options in the *new* and the *old* VA. This approach deliberately ignores the highly subjective and problematic aspects in forecasting whether one particular investment, asset class or product will outperform or 'do better' than another. Instead we assume that all investments earn the same risk-adjusted rate and focus on the discounted value of the downside protection and investment guarantees. This approach to valuation is widespread in the capital markets and we simply borrow these ideas within the context of variable annuities. We illustrate our thinking with a variety of case studies and also draw some general conclusions about the conditions under which a VA exchange is likely to be suitable.

Note that while the mathematical algorithms underlying these calculations might not be accessible (or of interest) to a wide range of planners, the authors have developed a basic software implementation that is available on the journal's website and can be downloaded to analyze the embedded options. From a pedagogical perspective, simply toggling the input parameters and viewing the results yield some interesting insights.

EXISTING LITERATURE AND RESEARCH

Parallel to the growth of variable annuities themselves, academic and scholarly research in this area has grown considerably. Reichenstien (2000) and Toolson (1991) analyze the effect of time horizons, investment returns, added fees and marginal tax rates on the relative appeal of variable annuities compared to mutual funds. Other works on the costs and benefits of nonqualified variable annuities compared to mutual funds include Mawr (1996), Price Waterhouse Coopers (1997), Milevsky and Panyagometh (2001), Ding and Peterson (2003), among others. Grote (2003) argues that the increasing attractiveness of annuities is due to increased client longevity, anticipated single-digit equity returns in coming years, the risk of unsustainable levels of portfolio withdrawals, more product flexibility and a continued decline in defined benefits plans. Kahn (2000) examines whether or not tax-deferred variable annuities are appropriate for an investment portfolio relative to mutual finds. He summarizes that the decision to purchase a tax-deferred variable annuity should be carefully tailored to the individual's needs, goals and pre-existing portfolio. The debate around the suitability of variable annuities is also documented by Newton (1999). In a recent issue of the *Journal of Financial Planning*, Duff (2003) discussed situations where investors should and should not exchange their annuities.

In sum, a number of researchers have examined this market in detail, and our paper follows along the same lines of research by attempting to provide some analytic guidance based on the probability one product will provide more value than the other.

[4] According to the NASD alert (http://www.nasdr.com/alert_annuityexchanges.htm), "…You should exchange your annuity only when you determine, after knowing all the facts that it is better for you and not just better for the person who is trying to sell the new contract to you…"

ANALYSIS

Our main qualitative proposition is that a VA policy consists of a portfolio of embedded financial guarantees (a.k.a. options) in the following order of importance and priority.

1. **Lapse value.** This captures the cash-flow that comes from liquidating or surrendering the policy – possibly after paying a contingent deferred surrender charge (CDSC) – and is tied to the evolution of the market value of the sub-accounts. Most investors purchase VAs exclusively for their lapse value since they view the instrument as a (tax-preferred) savings vehicle. Therefore, when contemplating an exchange, the policy holder should be cognizant of the *discounted lapse value* (DLV*)* of the new policy. All else being equal, the greater the contingent deferred surrender charges (CDSC) on the *new* policy and/or the greater the Mortality and Expense Risk Fees (from here, abbreviated M&E Fee) relative to the *old* policy, the lower the *discounted lapse value,* and the less likely it is that an exchange will add economic value to the investor.

2. **Death value**. Most VA policies contain some form of guaranteed minimum death benefit. This guarantee comes in various shapes and flavors, but usually consists of a return-of-premium guarantee together with a minimally guaranteed interest rate and/or anniversary step-up feature, which periodically raises the minimum guarantee to market value. And, while these guaranteed minimum death benefits (GMDB)[5] (as they are collectively known) have become a recent nuisance to the risk management and treasury departments within the insurance companies issuing these policies, the fact is that most consumers are more likely to lapse their VA than die and cash-in on the GMDB. Nevertheless, this embedded option must be priced in any transaction and we use the term *discounted mortality value* (DMV*)* to denote the current value of this guarantee.

3. **Guaranteed Annuity Rates**. All variable annuity policies contain an option to annuitize – in the form of a guaranteed mortality table and interest rate -- which is poorly understood and usually ignored. Many older VAs contain annuity rate guarantees that are linked to outdated mortality tables and are thus more valuable, relative to updated mortality tables that assume a longer life expectancy. More recent VA policies offer optional guaranteed minimum income benefit (GMIB) riders which are a form of guaranteed annuity rate. In general, we use the term *discounted annuity value* (DAV) to denote the collection of options associated with the ability to annuitize the policy at a pre-specified mortality table and interest rate.

DECOMPOSING THE OPTIONS

Switching one policy for another is an exercise in comparing embedded option values. We denote and model the value of the exchange with our main equation:

$$Exchange\ \ Value = (DLV_{new} + DMV_{new} + DAV_{new}) - (DLV_{old} + DMV_{old} + DAV_{old}) .$$

Every VA contains a financial component related to the *lapse* value, a guarantee associated with the *death* benefit and a guarantee associated with the payout *annuity* benefit. These dollar values are denoted by *DLV*, *DMV* and *DAV* respectively. The *old* and *new* values are subscripted on these

5 Guarantted minimum death benefit (GMDB) can be viewed as an exercise price of a put option. If the market value of the VA is less than the GMDB, the VA holder has a option to sell the VA back to the insurer at the GMDB.

symbols. A particular transaction or exchange might result in a reduction in the lapse value component *DLV*, but a significant increase in the mortality component *DMV,* thus making the entire transaction profitable and value enhancing. Overall, our analytic approach is to add-up the value of the options that are being surrendered, and compare them to the value of the options that are being acquired. If the former is less than the latter, the transaction is suitable on *economic* grounds. Otherwise, it destroys economic value.

Most, if not all of the recent innovations and riders in the variable annuity market can be placed within these three option/guarantee categories. For example, guaranteed minimum accumulation benefits (GMAB) that are not tied to mortality or annuitization -- and simply specify a minimal maturity guarantee -- would be placed in the discounted lapse value category. Enhanced death benefits in the form of ratchets and roll-ups would form part of the discounted mortality value calculations. Likewise, guaranteed minimum withdrawal benefits (GMWBs) would come under the lapse value category, etc.

The details of computing the option value for each of the components is briefly described in the appendix to this paper, while greater mathematical detail is available in the companion technical paper by Milevsky and Panyagometh (2003). For now, we present a number of case studies using the above-mentioned software that will help explain the process.

CASE STUDIES AND EXAMPLES

Case Study #1: Alex.

In our first case study, we assume that Alex is a 50 year-old female. She currently has exactly $250,000 (market value) invested in a variable annuity policy. The funds are allocated amongst a large collection of investment sub-accounts, but the bulk of her funds are in the value and growth equity asset classes. The investment volatility as measured by the standard deviation of returns of these sub-accounts is approximately 18%, which, according to Ibbotson Associates, is consistent with long-run volatility of North American equity markets. Alex purchased this VA a little more than five years ago, and if she were to lapse or exchange her VA policy today, she would face a contingent deferred surrender charge (CDSC) of 2.7% of the $250,000 market value. Figure #1 plots a typical CDSC schedule, which we created based on averages of CDSC schedules across the universe of VA policies tracked by Morningstar. The Mortality and Expense (M&E) fee on her current VA is 100 basis points per annum (withdrawn monthly), and the guaranteed minimum death benefit (GMDB) is $210,000. Thus, regardless of how markets perform over the next few years, if Alex dies while still owning the VA, her estate is guaranteed to receive at least $210,000. And, while that sum is clearly less than the current market value of $250,000, there is a chance markets will decline at some point in the future, and therefore, there is a chance that at the time of her death, her account value will be less than $210,000, in which case the guarantee will 'kick it'. Of course the probability of this event is quite remote and thus the value of this guarantee is likely minimal, but nevertheless, it is part of the options package. In the language of option pricing, her GMDB is currently 16% *out of the money* because the market value far exceeds the guaranteed death benefit. This is most likely due to her original purchase policy of five years grown since then.

Exhibit #1 Placed Here

Now, let's assume that Alex has the opportunity to exchange her existing VA for a new product that would automatically step-up her death benefit to the current market value, which would be $243,250 (the market value of the old VA minus the 2.7% surrender fee equals $250,000 - $6,750). The *new* VA would then have an M&E fee of 120 basis points, and the surrender charge schedule would start anew at 5% in the initial year, and decline to zero between the 7^{th} and the 11^{th} year.

Exhibit #2 Placed Here

As the reader can see from Exhibit #2, exchanging the old VA for the new VA will result in an economic loss of $10,005 for Alex. This can be broken down into a loss of $297 from the discounted mortality value and a loss of $9,708 from discounted lapse value implicit in starting the CDSC schedule anew. On a net basis, this transaction results in a loss of $10,005, and the exchange will *not* add any economic value. The culprit in this negative-valued proposition is likely the 2.7% CDSC (=$6,750) that must be paid immediately, and exceeds the value of any step-up in guaranteed death benefit basis. Recall that Alex is 50 years old, and the probability of survival during the next 10 to 15 years is quite high. Once lapses are taken into account, the probability of ever using the death benefit is relatively low. Note that we have implicitly assumed the guaranteed annuity rates (GAR) are the same on both contracts – or that Alex has no intention of ever annuitizing -- which is why the values are listed as N.A. in the bottom of Exhibit #2. If the GAR parameters were non-trivial, we would add a third component to the package.

Case Study #2: Paul

In our second case study we take Paul who is 55 years old. Most of the variables in the previous case remain the same here, however in contrast to Case Study #1, the guaranteed minimum death benefit (GMDB) on Paul's current VA is $190,000. Also, the new VA he is contemplating purchasing has a higher M&E fee of 130 basis points, but also offers a 1% bonus credit on the purchase premium. Thus, if Paul were to exchange his current VA with a market value of $250,000, the opening balance on the new VA would be $245,683 (old VA – surrender fees + bonus credits = $250,000 - $6750 + $2433 = $245,683), and the guaranteed minimum death benefit would automatically be stepped up to the $245,683 value.

Exhibit #3 Placed Here

As noted in Exhibit #3 which summarizes this case, exchanging the old VA for the new VA will result in losses of both discounted surrender value and discounted mortality value. In totality, this transaction results in a discounted economic loss of $10,027. The implicit benefits of bonus credits and the step-up in basis is less than the implicit cost of a higher M&E fee and the cost from resetting the surrender charge schedule anew at 5%. Note the difference between the discounted lapse value on the *old* VA for 50 year-old Alex, which is $177,336 compared to the discounted lapse value for 55 year-old Paul, which is $165,481. The gap of close to $12,000 is due to the fact that Paul is more likely to die within the non-lapsed 'life' of the VA compared to Alex, and therefore the lapse value (which is conditional on being alive) is lower. Using the same reasoning, the discounted mortality value is higher for Paul, who is older ($46,149), compared to Alex ($34,021). The sum of the two components is close to the same value for either Alex ($211,356) compared to Paul, ($211,629). Another important insight is that for either Alex or Paul, the total value of the options package is far less than the $250,000 market value of the VA. This immediate loss may seem odd at first glance, but is a direct result of the

M&E fees that are paid as well as the probability of lapsation, both of which impose unavoidable fees on the account, and reduce the present value of the total options package.

Case Study #3: Mary

In this case we consider 65-year-old Mary. As in the previous cases, the variables remain the same, except that the guaranteed minimum death benefit (GMDB) on her current VA is a mere $150,000. Mary has the option to exchange her existing VA for a new VA. The new VA has a higher M&E fee of 110 basis points, but offers a 2% bonus credit. If Mary opts to switch her existing VA, the account market value of the new VA will be $248,115 (the market value of the old VA – surrender fees + bonus credits = $250,000 - $6750 + $4,865 = $248,115).

Exhibit #4 Placed Here

According to Exhibit #4, a loss of $2,894 from the discounted lapse value will be approximately offset by a gain of $2,911 from the step-up in basis of the VA and the discounted mortality value. Her advanced age relative to Alex and Paul adds more to the discounted mortality value. In sum, exchanging the old VA for the new VA will result in an economic gain of $18 per $250,000 invested in the VA. Of course, this trivial sum far exceeds the hassle costs of doing the transaction, but the point is that one could make a strong case for exchanging the VA even though a CDSC is paid and a higher M&E is imposed on a purely economic basis. In practice, *total values* in the mere hundreds of dollars – either positive or negative -- on a VA whose market value is in the hundreds of thousands should be taken in context. It is only once the magnitude reaches the thousands, or tens of thousands as in the case of Alex and Paul, that our analysis should be taken as a meaningful indication of a substantial change in economic welfare.

Case Study #4: Peter

Lastly, we consider a 60 year-old male whom we call Peter. The variables in this case remain the same with the exception that the guaranteed minimum death benefit (GMDB) on his current VA is a $100,000 and the new VA offers 7% bonus credits of purchase payment. Note that in this case, the new VA has a higher M&E fee of 110 basis points. As a result of switching to the new VA, the account market value of the new VA will be $260,278 (the market value of the old VA – surrender fees + bonus credits = $250,000 - $6750 + $17,028 = $260,278), and the guaranteed minimum death benefit will automatically be stepped up to this market value.

Exhibit #5 Placed Here

As the reader can see, exchanging the old VA for the new VA will result in a financial economic gain of $9,201 per $250,000 invested in the VA. This can be broken down into a gain of $3,834 from the surrender value and a gain of $5,367 from mortality value, thus adding close to $10,000 in economic value from the VA exchange.

GENERAL OBSERVATIONS

Exhibit #6 presents a number of additional scenarios for VA exchanges using a variety of different minimum guaranteed death benefit (GMDB) levels and Mortality & Expense (M&E) insurance fees. In general we note the following robust observations:

Exhibit #6 Placed Here

- The variable annuity exchange with a 3% exit penalty (CDSC) was not profitable in any of the 15 cases illustrated in Situation #1. The immediate cost of a 3% exit penalty and the implicit cost associated with resetting the CDSC schedule anew at 5% is far greater than the benefit from the step-up in the death benefit and the benefit from a reduction in M&E charges.

- In the second situation in Exhibit #6 where the individual was not subject to any surrender charges or penalties, 5 of the 15 exchanges were profitable. From this we can see that when the M&E charge remains the same, an exchange is profitable if the old GMDB was 80% or greater of the market value. In this case, the benefit from the step-up in the death benefit is greater than the cost associated with resetting the CDSC schedule anew at 5%. Moreover, with a lower M&E charge of 80 basis points, an exchange would add value even without the benefit from the step-up (the old GMDB = 100%). The benefit from a reduction in M&E charges is greater than the cost associated with resetting the CDSC schedule.

- The third situation in Exhibit #6 illustrates that the variable annuity exchange resulting in a 5% exit penalty (CDSC) is not profitable in any of the 15 cases. We can further summarize this by saying that, switching a VA later in life from a VA at a lower or no exit penalty (CDSC) will have a higher probability of creating economic value compared to switching a VA earlier on in life at a higher exit penalty.

WHAT HAVE WE IGNORED?

Our analysis has focused entirely on the economic suitability of the *exchange*, as opposed to the suitability of the annuity *per se* and the need to match financial products to the actual needs of the consumer. *The issues surrounding the latter question are much more complicated than the factors impacting the former*. For example, we have intentionally steered clear of the contentious debate of whether variable annuity policies are preferred to mutual funds from a tax perspective, or whether variable annuities should be placed within qualified plans since their gains are already tax-deferred. For example, recent tax rate changes in the U.S. have reduced the relative value of tax deferral embedded within variable annuities, and we refer the reader to Milevsky and Panyagometh (2001, 2003) and Milevsky and Posner (2001) for an in depth discussion of these issues.

Likewise, our case studies and examples have implicitly assumed that the credit ratings of the issuing companies are identical, and thus the exchange can be viewed purely from an option value perspective. In practice, if the new or the old policies differ in 'credit security', our analysis must be adjusted.

According to the Life Insurance Marketing Research Association (LIMRA) which conducted a 1997 study, less than 1% of variable annuity contracts were ever annuitized; which is consistent with actuarial reserving (or the lack thereof) for these liabilities. It seems that few consumers think of annuities as a source of lifetime income, and only use them as accumulation vehicles. In contrast to the low utilization rate, we refer the interested reader to the recent paper by Chen and Milevsky (2003) for a discussion on the importance of longevity insurance *vis a vis* payout annuities and the value of the embedded option to annuitize. This feature will likely grow in popularity as the baby-boomers approach their 'retirement income' years, which is why we believe all *three* embedded options should be discussed when buying or exchanging VAs.

CONCLUSION

The $800 billion variable annuity market seems a mystery to many observers, and the recent explosion of added bells and whistles has not helped matters. In fact, an apocryphal story claims that New York's ex-governor Mario Cuomo started a keynote address to members of NAVA by stating that despite many attempts to educate him, he *still* had no idea what a variable annuity actually was, and that he would therefore speak about other matters.

We argue that the variable annuity package can be decomposed into three distinct financial guarantees or options. The first relates to the possibility of lapsation, the second relates to mortality and the third relates to annuitization. In total, these options can be analyzed and compared using conventional techniques. As a byproduct, we offer an impartial method for determining whether a particular exchange is (or was) economically suitable. Our methodology is based on the financial theory of option pricing, and is actually at the heart of risk management practices employed by insurance companies to manage these same guarantees. Insurance companies are well aware – or at least *should* be aware – of the value embedded in each one of these added riders. Most likely they have to set aside reserves and capital for these guarantees. We are reversing the process and advocating that policyholders use these same risk management techniques to access if and when a particular exchange has added value.

From a technical point of view, this paper has computed the risk-neutral expected payoff (a.k.a the NPV) from the embedded options, and then compared them across the two products. In theory, one could go a step further and compute the *entire distribution* of payoffs across both products, and then display the *probability* that an exchange adds economic value, as opposed to whether it does on an expected value basis. We leave this to future research.

In sum, regardless of precise methodology, we believe that an objective numerical algorithm *can* provide reassuring guidance and perhaps even legal justification for 'VA swapping' especially given the recent heightened awareness regarding regulatory issues and suitability concerns.

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TECHNICAL APPENDIX

We focus our attention on the Discounted Lapse Value (DLV) and the Discounted Mortality Value (DMV) to illustrate the main analytics in this appendix. The third embedded option, which is the Discounted Annuity Value (DAV) can be treated similarly, but we omit the discussion to conserve space. Our model assumed the pre-lapsed market value of the VA policy follows a continuous-time stochastic process denoted by the symbol U_t, with a growth rate of (μ-q) and a volatility of σ, where q denotes the M&E fee paid continuously in time. If and when the policy is exogenously surrendered, lapsed or terminated -- but not upon death -- it will pay-off an amount (1 - $k(t)$)U_t, where $k(t)$ denotes the Contingent Deferred Surrender Charge (CDSC) percentage, applicable at time t. Exhibit #1 is an illustration of a typical CDSC curve which starts close to 5% and then declines to zero after seven years.

We model mortality and the future lifetime random variable **T** via a Gompertz-Makeham hazard rate – see Milevsky & Posner (2001) for an explanation of this model -- denoted by λ(s). The probability of surviving to any given age is:

$$\Pr\left[\mathbf{T}_x \geq t\right] = e^{-\int_0^t \lambda(x+s)ds} = 1 - F_x(t) \qquad (1)$$

where the last term represents the cumulative distribution function (CDF) of the future lifetime random variable. The probability density function (PDF) of the future lifetime random variable is $f_x(t) = F_x^{'}(t)$, where the prime symbol denotes a derivative with respect to time.

The exogenous future lapse-time random variable **L** will be modeled on the hazard rate function γ(t), with a PDF denoted by g(t) and a CDF denoted by G(t). Both are defined as in the mortality case, so that G(t) = Pr[**L** ≤ t] and 1 - G(t) denotes the probability of not lapsing before time t, while 1 - F_x(t) denotes the probability of not dying before time t — both under an initial age of x. Finally, the general formula for the Discounted Lapse Value (DLV) used in the paper is:

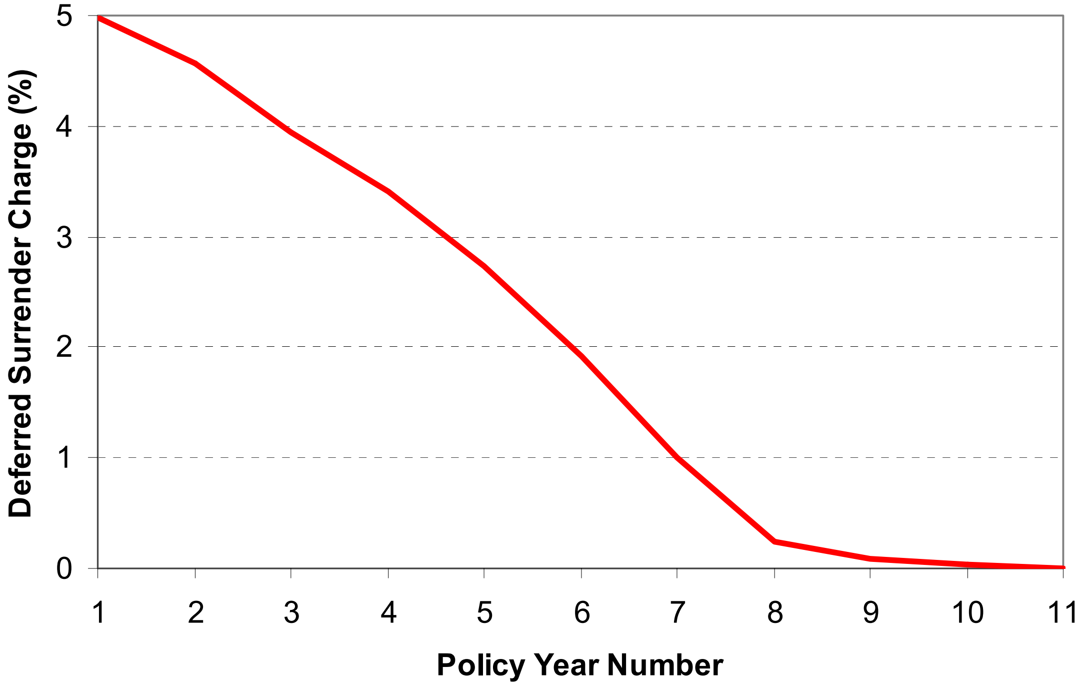
$$DLV = u\int_0^\infty e^{-qt}(1-k(t))(1-F_x(t))g(t)dt \qquad (2)$$

Now let the function $\phi(u,v,\sigma,r,q,t)$ denote the classical Black-Scholes-Merton put option price where *u* denotes the current market value of the underlying account, *v* denotes the strike price, σ denotes the investment return's volatility, *r* denotes the risk-free rate, *q* denotes the dividend yield (which in our case is the M&E fee) and *t* is the maturity of the option. The Discounted Mortality Value (DMV) of the VA policy is the combination of the account value — payable at death — plus any guaranteed portion discounted for lapsation and mortality. Mathematically this can be expressed as:

$$DMV := \int_0^\infty (u+\phi(u,v,\sigma,r,q,t))(1-G(t))f_x(t)dt \qquad (3)$$

In sum, a variable annuity policy can be viewed as the linear sum of three option-like quantities. The value of each embedded option within the VA package is computed by *integrating* (read: adding up) the probabilities the option will actually be exercised at a given point in time, multiplied by the discounted payoff from the option, if indeed it is used.

Exhibit #1: Contingent Deferred Surrender Charge (CDSC) Schedule



Note: Our methodology requires a contininuous CDSC schedule which can be integrated against the future market value of the variable annuity at the random time of lapsation. We have estimated this curve based on asset-weighted average numbers provided by Morningstar Principia Pro, as of December 2002.

Exhibit #2: Output from case study one.

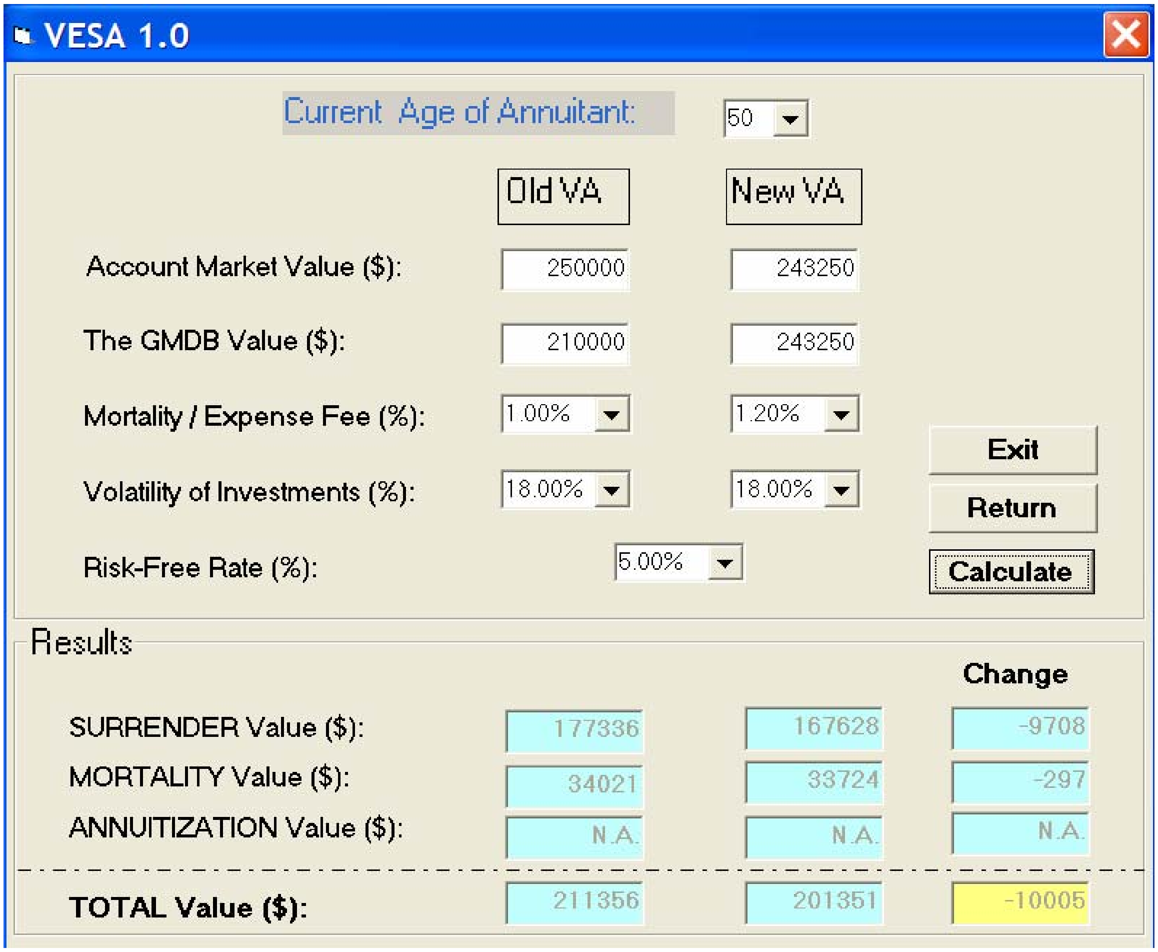


Exhibit #3: Output from case study two.

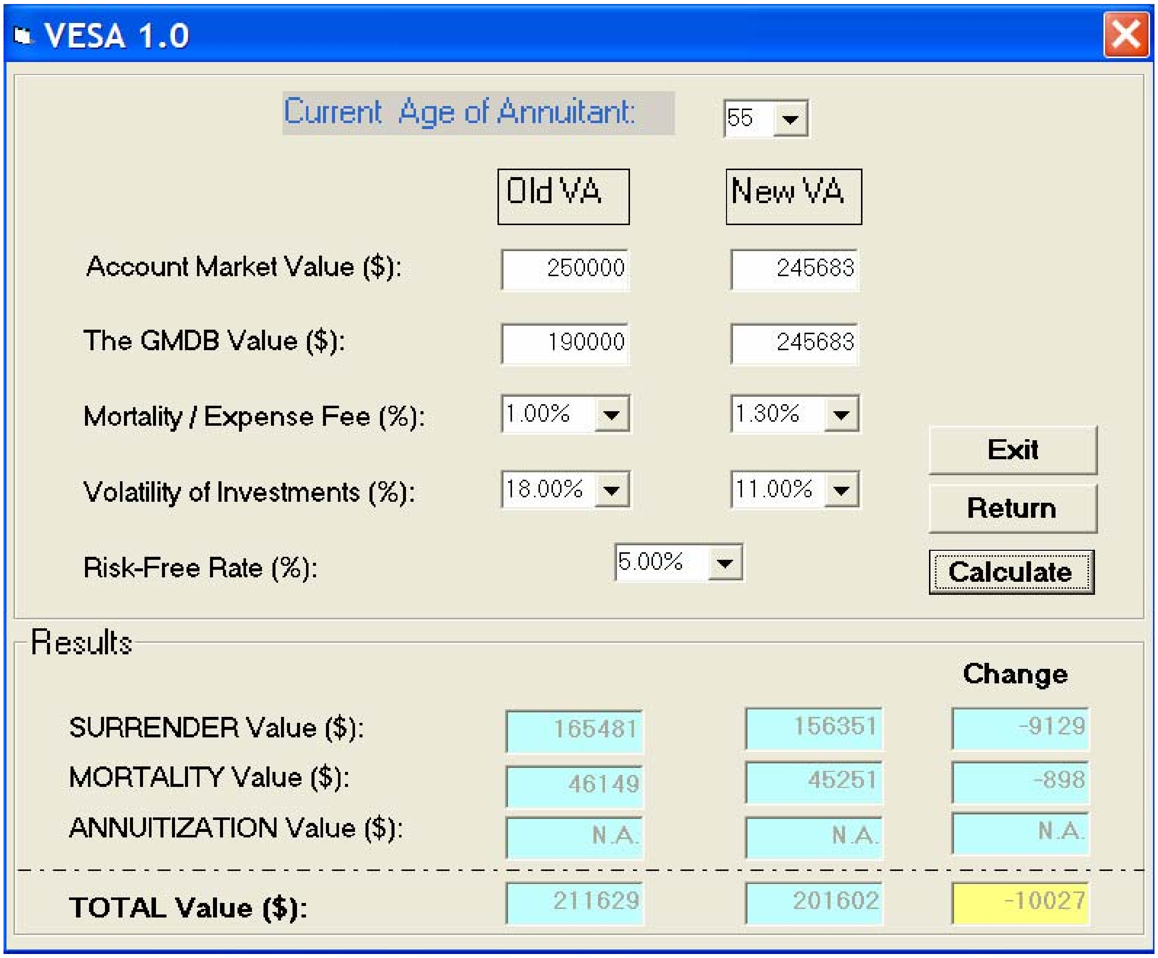


Exhibit #4: Output from case study three.

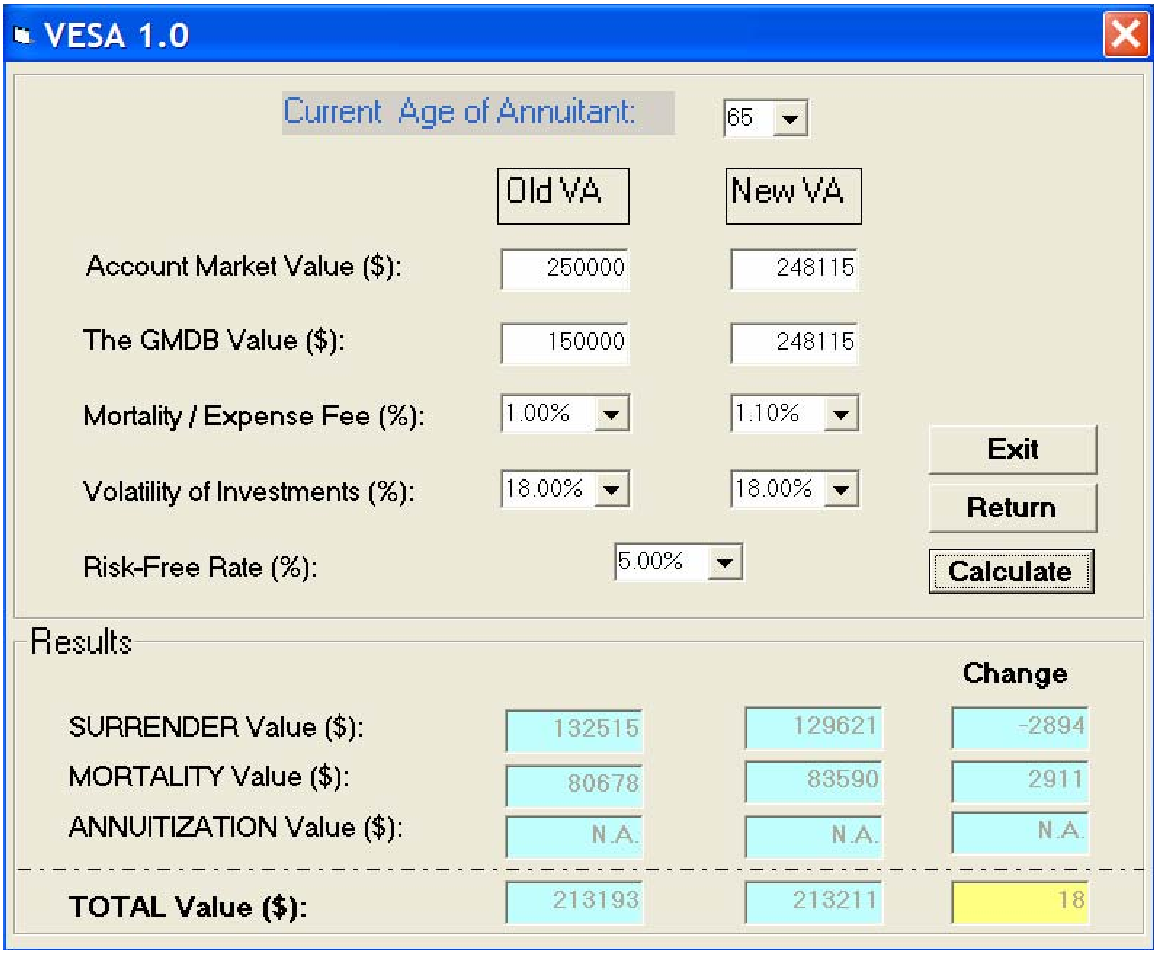


Exhibit #5: Output from case study four.

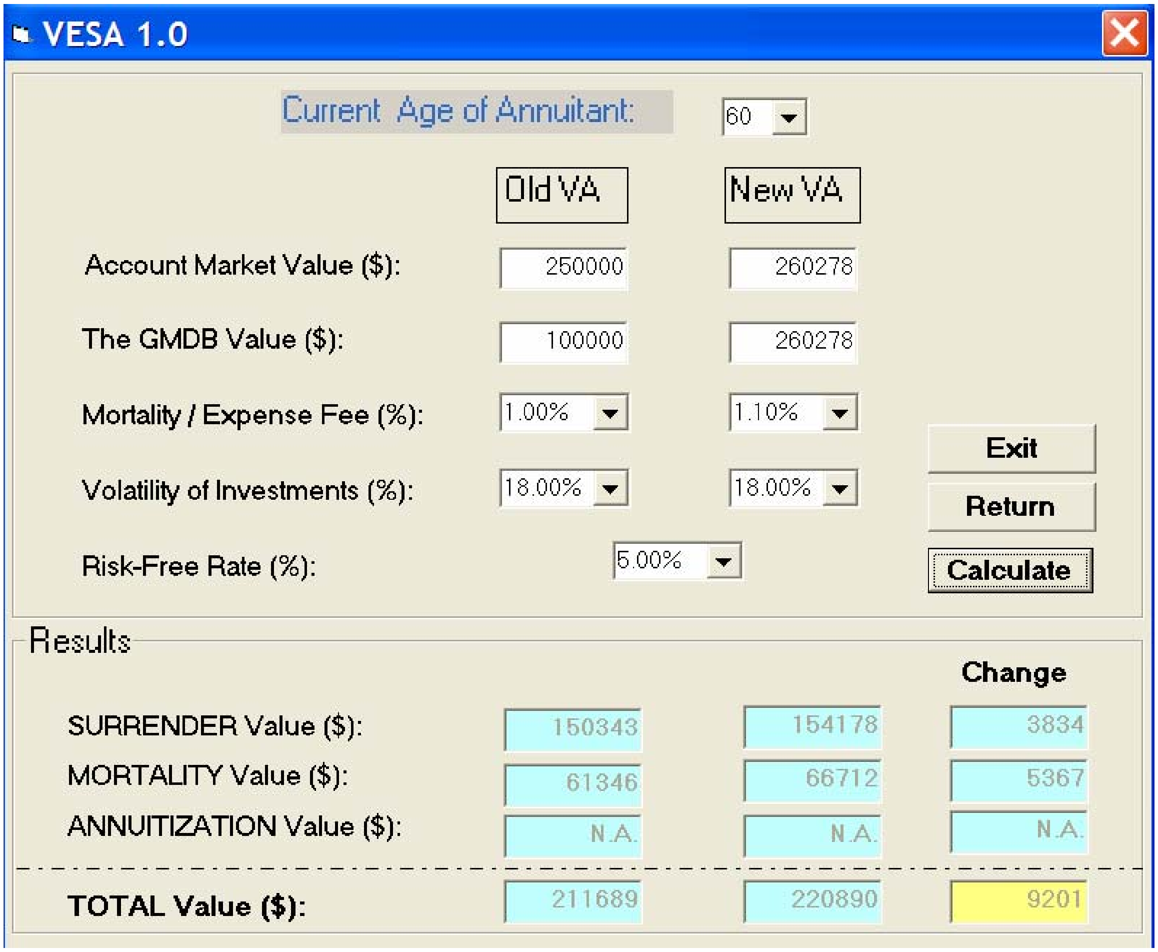


Exhibit #6: Additional examples.

*How Much Do You Gain or Lose from Exchanging the VA?**			
Situation #1: You are 60 years old, your VA is worth $100,000 and you have to pay a 3% CDSC penalty to exit			
Old GMDB as % of Market Value	**New M&E Fee (Basis Points)**		
	80	**100**	**120**
60%	-567	-1,711	-2,816
80%	-916	-2,060	-3,166
100%	-1,643	-2,787	-3,893
120%	-2,858	-4,002	-5,108
200%	-10,505	-11,649	-12,755
Situation #2: You are 60 years old, your VA is worth $100,000 and there is no CDSC penalty for exiting			
Old GMDB as % of Market Value	**New M&E Fee (Basis Points)**		
	80	**100**	**120**
60%	1,716	537	-603
80%	1,367	187	-952
100%	639	-540	-1,680
120%	-576	-1,755	-2,895
200%	-8,223	-9,402	-10,542
Situation #3: You are 75 years old, your VA is worth $100,000 and you have to pay a 5% CDSC penalty to exit			
Old GMDB as % of Market Value	**New M&E Fee (Basis Points)**		
	80	**100**	**120**
60%	-1,316	-1,652	-1,975
80%	-2,131	-2,467	-2,790
100%	-4,176	-4,512	-4,835
120%	-7,985	-8,320	-8,643
200%	-31,878	-32,214	-32,537

* Assumptions: the M&E fee of the *old* VA is 100 basis points per annum. The surrender charge schedule of the *new* VA would start at 5% in the initial year, and (decline to zero between the 7th and 11th year. The investment volatility of VA sub-accounts is 18%, and the risk-free rate is 5%